Measuring the Earth’s Magnetic Field

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Abstract

In this experiment, the Earth’s magnetic field is measured. Using a Helmholtz coil, a calibration constant is determined that is applied to deflection values obtained by a ballistic galvanometer. The determined values for the vertical, perpendicular, and parallel components of the Earth’s magnetic field are, respectively: 

- Vertical: $-38056 \pm 732$ nT,
- Perpendicular: $-28024 \pm 521$ nT,
- Parallel: $26700 \pm 607$ nT.

The horizontal component of the Earth’s magnetic field was determined to be $38708 \pm 604$ nT, and finally the total magnetic field of the Earth was determined to be $54282 \pm 658$ nT.
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Introduction

The Earth’s magnetic field has been thoroughly studied for hundreds of years. Carl Friedrich Gauss measured its strength in 1835, and from subsequent measurements it has been determined that it has a relative decay of 10% over the past 150 years. Early studies were conducted for the purposes of navigation. In fact, compasses have been used since the 11th century A.D. Perhaps the most important effect of the Earth’s magnetic field is that it protects the Earth from violent solar winds. Without a strong enough magnetic field to deflect these incoming charged particles the Earth’s ozone layer would be stripped, leaving Earth to an outcome similar to that of Mars.

The experiment described in detail in this report was conducted to measure the intensity of the Earth’s magnetic field at the specific location in which the experiment took place. An Earth inductor is used to perform the various components of the experiment. It consists of a coil of wire with a specific amount of turns (how many times the coil is wrapped around) and a specific radius. It is connected to a ballistic galvanometer (a sensitive instrument that measures electric current), and can be flipped about an axis to induce an EMF (Electromotive Force). This induced EMF is measurable by the deflection in the ballistic galvanometer. The amount of deflection is proportional to the component of Earth’s magnetic field that is perpendicular to the Earth inductor. For reliable and comparable data, a calibration is necessary to determine the proportionality constant. In this experiment, calibration was achieved using a Helmholtz coil with a known magnetic field. A Helmholtz coil consists of two identical coils of wire separated by a distance that is equal to the radius of the coils. They are commonly used in experiments to produce a region that has a virtually uniform magnetic field.

The following sections of this report will consist of the Background, Experimental Techniques, Data, Analysis, Discussion, Conclusion, Appendix, and References. Each section has been carefully composed to provide a depth of knowledge pertaining to the data recorded, calculations, and error analysis.

1 (Earth's Magnetic Field)
Background
When the Earth inductor coil is flipped in a magnetic field an EMF is produced that is described by the equation:

$$\mathcal{E} = n \frac{d\Phi}{dt} \quad (1.0)$$

Where, in the above equation, $\mathcal{E}$ is the EMF, $n$ is the number of turns, $\Phi$ is the flux through the coil, and $t$ is time.

The above EMF must equal the potential drop around the circuit, so:

$$n \frac{d\Phi}{dt} = i_t R_1 \quad (1.1)$$

Where, in the equation above, $i_t$ is the current in the circuit, and $R_1$ is the resistance.

The total charge ($Q$) that flows into the galvanometer when it is flipped is determined by integrating over the duration of the flip:

$$Q = \int i_t dt = \frac{n}{R_1} \Delta \Phi \quad (1.2)$$

For a 180° flip:

$$\Delta \Phi = 2B_\perp A \quad (1.3)$$

Where, in the above equation (1.3), $B_\perp$ is the component of the Earth’s magnetic field that is perpendicular to the coil, and $A$ is the area of the coil. Combining equations (1.2) and (1.3) we have:

$$Q = \frac{n}{R_1} * 2B_\perp A \quad (1.4)$$

The deflection of the ballistic galvanometer is proportional to the charge that flows through it:

$$d = KQ \quad (1.5)$$

Where, in the above equation (1.5), $d$ is the amount of deflection, and $K$ is a proportionality constant with respect to the galvanometer. Combining equations (1.5) and (1.4) we now have:

$$Q = \frac{d}{K} = \frac{n}{R_1} * 2B_\perp A \quad (1.6)$$

Equation (1.6) can also be expressed as,
Where, in the above equation (1.7), the component of the Earth’s magnetic field is related to the deflection of the galvanometer. Since \( K \) is unknown to us, we must find it in order to make use of this formula. Since \( K \) is the proportionality constant for the galvanometer, a calibration process must be required. A Helmholtz coil is used to create a known magnetic field, and the effect of the flip coil is then measured in the total magnetic field (Earth’s field and Helmoltz’s field). The following algebraic equations describe the relations:

\[
B_T = B_E + B_H \quad (2.0)
\]

Where, in the above equation, \( B_T \) is the total magnetic field, and \( B_E \) and \( B_H \) are the magnetic fields of the Earth and the Helmholtz coil, respectively. The deflection of the galvanometer follows the same additive relationship:

\[
d_T = d_E + d_H \quad (2.1)
\]

Where, in the above equation, \( d_T \) is the total deflection, and \( d_E \) and \( d_H \) are the deflections due to the Earth and the Helmholtz coil. Equations (2.0) and (2.1) can be combined with (1.7) to produce a set of relations:

\[
B_T = d_T \frac{R_1}{nAK} = (d_E + d_H) \left( \frac{R_1}{nAK} \right) \quad (2.2)
\]

It can also be shown that,

\[
B_E = d_E \frac{R_1}{nAK} \quad (2.3)
\]

The magnetic field of the Helmholtz coil would then be:

\[
B_H = B_T - B_E = (d_T - d_E) \left( \frac{R_1}{nAK} \right) \quad (2.4)
\]

Equation (2.4) then leads us to the equation for the Earth’s magnetic field:

\[
B_E = B_H \frac{d_E}{d_T - d_E} \quad (2.5)
\]

Let \( C = B_H/(d_T - d_E) \) \quad (2.6)

\[
B_E = Cd_E \quad (2.7)
\]
Where, in the above equation (2.6), C is the “calibration” constant that can be determined from the calibration measurements using the deflection measured for the Earth’s vertical field component. The constant can then be applied to the other components of the Earth’s magnetic field. For each non-zero value of the current, I, the measurement of the total deflection, \(d_T\), results in a measurement of the calibration constant, C, using the deflection measured for the Earth, \(d_E\), when the current was zero.

We need to calculate the uncertainty associated with the value, so using equation A.1 from the appendix:

\[
\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \cdots + \left(\frac{\partial f}{\partial n}\right)^2 \sigma_n^2} \quad (A.1)
\]

\[\sigma_{B_E} = \sqrt{\left(\frac{\partial B_E}{\partial C}\right)^2 \sigma_C^2 + \left(\frac{\partial B_E}{\partial d_E}\right)^2 \sigma_{d_E}^2} \]

\[\sigma_{B_E} = \sqrt{(d_E)^2 \sigma_C^2 + (C)^2 \sigma_{d_E}^2} \quad (2.75)\]

Equation (2.6) can also be expressed as

\[d_T = \frac{B_H}{C} - d_E \quad (2.8)\]

This form makes it obvious that C and \(d_E\) can be obtained by plotting \(d_T\) versus \(B_H\). The slope of the best fit line through the points will then be \(1/C\) with \(d_E\) as the intercept.

The magnetic field of the Helmholtz coil is determined by the following formula:

\[B_H = \frac{8\mu_0 NI}{a\sqrt{125}} \quad (2.9)\]

Where, in the equation above, \(\mu_0\) is the permeability of free space, \(N\) is the number of turns in each coil, I is the current flowing through the coils in amperes, and \(a\) is the radius of the coils in meters. This equation can be derived by starting with the formula for the on-axis field due to a single wire loop (which is derived from the Biot-Savart law).\(^2\) The uncertainty in the above equation (2.9) can be found using the error propagation formula, Appendix A.1:

\(^2\) (Helmholtz Coil)
\[
\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \cdots + \left(\frac{\partial f}{\partial n}\right)^2 \sigma_n^2} \quad (A.1)
\]

In this case, the above equation reduces to the following:

\[
\sigma_{B_H} = \sqrt{\left(\frac{\partial B}{\partial I}\right)^2 \sigma_I^2 + \left(\frac{\partial B}{\partial a}\right)^2 \sigma_a^2}
\]

\[
\sigma_{B_H} = \sqrt{\left(\frac{8\mu_0 N}{a\sqrt{125}}\right)^2 \sigma_I^2 + \left(\frac{8\mu_0 N a}{a^2\sqrt{125}}\right)^2 \sigma_a^2} \quad (2.91)
\]

We also need to calculate the uncertainty for the calibration constant in (2.6) using the error propagation formula A.1 from the Appendix:

\[
\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \cdots + \left(\frac{\partial f}{\partial n}\right)^2 \sigma_n^2} \quad (A.1)
\]

For the case of equation (2.6), the above equation becomes:

\[
\sigma_C = \sqrt{\left(\frac{\partial C}{\partial B_H}\right)^2 \sigma_{B_H}^2 + \left(\frac{\partial C}{\partial d_T}\right)^2 \sigma_{d_T}^2 + \left(\frac{\partial C}{\partial d_E}\right)^2 \sigma_{d_E}^2}
\]

\[
\sigma_C = \sqrt{\left(\frac{1}{d_T - d_E}\right)^2 \sigma_{B_H}^2 + \left(-\frac{B_H}{(d_E - d_T)^2}\right)^2 \sigma_{d_T}^2 + \left(\frac{B_H}{(d_E - d_T)^2}\right)^2 \sigma_{d_E}^2} \quad (2.92)
\]

From our data, we can also determine the horizontal component of the Earth’s magnetic field:

\[
B_H = \sqrt{B_{||}^2 + B_{\perp}^2} \quad (3.0)
\]

Where, in the above equation, \(B_H\) is the horizontal component of Earth’s magnetic field, \(B_{\parallel}\) is the component that is parallel to the meridian, and \(B_{\perp}\) is the component that is perpendicular to the meridian. The uncertainty for the above equation can be found by using equation A.1 from the Appendix, and adapting it to our needs:
\[ \sigma_{B_h} = \sqrt{\left(\frac{\partial B_h}{\partial B_h} \right)^2 \sigma_{B_h}^2 + \left(\frac{\partial B_h}{\partial B_\perp} \right)^2 \sigma_{B_\perp}^2} \]

\[ \sigma_{B_h} = \sqrt{\left(\frac{B_\parallel}{B_\parallel^2 + B_\perp^2} \right)^2 \sigma_{B_\parallel}^2 + \left(\frac{B_\perp}{B_\parallel^2 + B_\perp^2} \right)^2 \sigma_{B_\perp}^2} \]

Which just becomes,

\[ \sigma_{B_h} = \sqrt{\frac{B_\parallel^2 \sigma_{B_\parallel}^2 + B_\perp^2 \sigma_{B_\perp}^2}{B_\parallel^2 + B_\perp^2}} \]  

(3.01)

Now, the total Earth magnetic field, \( B_{Earth\,Tot.} \), can be determined by:

\[ B_{Earth\,Tot.} = \sqrt{B_h^2 + B_V^2} \]  

(3.1)

Where, in the above equation (3.1), \( B_h \) is the horizontal component of the Earth’s magnetic field, and \( B_V \) is the vertical component. The uncertainty in the total Earth magnetic field can be calculated by using equation A.1 from the appendix, but we just formulated this same type of equation in (3.01), so similar to before:

\[ \sigma_{B_{Earth\,Tot.}} = \sqrt{\left(\frac{B_h}{B_h^2 + B_V^2} \right)^2 \sigma_{B_h}^2 + \left(\frac{B_V}{B_h^2 + B_V^2} \right)^2 \sigma_{B_V}^2} \]

\[ \sigma_{B_{Earth\,Tot.}} = \sqrt{\frac{B_h^2 \sigma_{B_h}^2 + B_V^2 \sigma_{B_V}^2}{B_h^2 + B_V^2}} \]  

(3.11)

To find the average C value we can use the formula for the simple arithmetic average (mean):

\[ mean = \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \]  

(3.2)
The standard deviation is then:

\[ \sigma_x = \sqrt{\frac{\sum_{i=1}^{N} (\bar{x} - x_i)^2}{N - 1}} \quad (3.3) \]
Experimental Techniques

Diagrams and Images

**Image 1:** An Earth inductor (flip coil). This is similar to the type of flip coil used in the experiments performed in this lab. This is an old Cenco model from the Greenslade Collection.\(^3\)

![Image of Earth inductor](image)

**Diagram 1:** This diagram depicts a Helmholtz coil. The radius distance is equal to the spacing of the coils. Current flows in through one coil and out at the bottom of the other. The magnetic field is uniform in the x-direction.\(^4\)

For the setup, the laboratory table was aligned with the wall, so that it had a known orientation with respect to the lab. The flip coil was connected to the galvanometer through a current limiting series resistor of 700 ohm resistance. The value of the resistance is to be chosen.

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\(^3\) (The Earth Inductor, or Delzenne's Circle)

\(^4\) (Hellwig, 2005)
such that the deflection of the galvanometer is within range. The components of the Earth’s magnetic field are then to be measured by flipping the coil. At each flip of the coil, the peak of the galvanometer deflection is recorded. This is done at least five times for each of the three components of the Earth’s magnetic field vector. The components to be measured are with the coils axis vertical, parallel to the meridian (North-South), and perpendicular to the meridian (East-West).

The second part of the experiment consisted of a calibration process in order to obtain a calibration constant. The calibration process used involved a Helmholtz coil. The Helmholtz coil creates a known magnetic field, and thus adds to the magnetic field of the Earth. To begin, the flip coil is placed inside the Helmholtz coil with the flip coils axis vertical. The Helmholtz coil was then connected to the current controlled power supply. Then, 27 deflection values were measured at various current values that were chosen to give a range of deflection values primarily near the high ends of the range. The radius of the Helmholtz coil was also measured (0.34 ± 0.005 m), and the number of turns was recorded (72 in each coil).
Data

Table 1: This table lists the data for first part of the experiment which consisted of various orientations of the flip coil in solely the Earth’s magnetic field. $d_E$ is the deflection of the galvanometer due to the Earth’s magnetic field, and $\Delta(d_E)$ is the associated uncertainty. The averages of the values and the standard deviations are provided for each orientation. The uncertainties are estimated.

<table>
<thead>
<tr>
<th>Vertical Orientation</th>
<th>Perpendicular Orientation (E-W)</th>
<th>Parallel Orientation (N-S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial</td>
<td>$d_E$ (m)</td>
<td>$\Delta(d_E)$ (m)</td>
</tr>
<tr>
<td>1</td>
<td>0.1670</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
<td>0.1665</td>
<td>0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.1670</td>
<td>0.002</td>
</tr>
<tr>
<td>4</td>
<td>0.1680</td>
<td>0.002</td>
</tr>
<tr>
<td>5</td>
<td>0.1680</td>
<td>0.002</td>
</tr>
<tr>
<td>Avg.</td>
<td>0.1673</td>
<td></td>
</tr>
<tr>
<td>Std.</td>
<td>3.354E-4</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: This table lists the data for the calibration part of the experiment in which data was recorded when using the known magnetic field of a Helmholtz coil. Deflection values listed here are the total deflection values (corresponding to the sum of the Earth’s and the Helmholtz magnetic fields). The delta symbol represents the uncertainty in a value, which is an estimate. We must also remember that the deflection data is recorded in centimeters, but must be converted to meters for use in further calculations. There are 27 values in total.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>$\Delta($Current$)$ (A)</th>
<th>Deflection (cm)</th>
<th>$\Delta($Deflection$)$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.02</td>
<td>0.005</td>
<td>18.6</td>
<td>0.2</td>
</tr>
<tr>
<td>-0.04</td>
<td>0.005</td>
<td>20.1</td>
<td>0.2</td>
</tr>
<tr>
<td>-0.06</td>
<td>0.005</td>
<td>21.8</td>
<td>0.2</td>
</tr>
<tr>
<td>-0.08</td>
<td>0.005</td>
<td>23.6</td>
<td>0.2</td>
</tr>
<tr>
<td>-0.09</td>
<td>0.005</td>
<td>24.4</td>
<td>0.2</td>
</tr>
<tr>
<td>0.01</td>
<td>0.005</td>
<td>15.7</td>
<td>0.2</td>
</tr>
<tr>
<td>0.02</td>
<td>0.005</td>
<td>15</td>
<td>0.2</td>
</tr>
<tr>
<td>0.04</td>
<td>0.005</td>
<td>13.3</td>
<td>0.2</td>
</tr>
<tr>
<td>0.06</td>
<td>0.005</td>
<td>11.6</td>
<td>0.2</td>
</tr>
<tr>
<td>0.08</td>
<td>0.005</td>
<td>9.95</td>
<td>0.2</td>
</tr>
<tr>
<td>0.09</td>
<td>0.005</td>
<td>9.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.10</td>
<td>0.005</td>
<td>8.2</td>
<td>0.2</td>
</tr>
<tr>
<td>0.12</td>
<td>0.005</td>
<td>6.6</td>
<td>0.2</td>
</tr>
<tr>
<td>0.14</td>
<td>0.005</td>
<td>4.9</td>
<td>0.2</td>
</tr>
</tbody>
</table>
The average calibration constant is the sum of the calibration constant, and the weighted average calibration constant value. The weighted average calibration constant value is included at the very bottom is the average calibration constant value, the standard deviation of the uncertainty squared (C/σC²), and the weight values (1/σC²) for the calibration constant. Also included at the very bottom is the average calibration constant value, the standard deviation of the calibration constant, and the weighted average calibration constant value. The weighted average calibration constant is the sum of the C/σC² values divided by the sum of the individual weights. Note: The (T) units are Tesla.

<table>
<thead>
<tr>
<th>B_H (T)</th>
<th>σ_BH (T)</th>
<th>C (T/m)</th>
<th>σ_C (T/m)</th>
<th>Ind. Dev. Of C</th>
<th>C/σ_C²</th>
<th>Weight (1/σ_C²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.808E-06</td>
<td>9.537E-07</td>
<td>-2.037E-04</td>
<td>5.958E-05</td>
<td>7.721E-10</td>
<td>-5.737E+04</td>
<td>2.817E+08</td>
</tr>
<tr>
<td>-1.142E-05</td>
<td>9.668E-07</td>
<td>-2.253E-04</td>
<td>2.284E-05</td>
<td>3.716E-11</td>
<td>-4.320E+05</td>
<td>1.917E+09</td>
</tr>
<tr>
<td>1.904E-06</td>
<td>9.525E-07</td>
<td>-1.849E-04</td>
<td>1.055E-04</td>
<td>2.169E-09</td>
<td>-1.661E+04</td>
<td>8.986E+07</td>
</tr>
<tr>
<td>3.808E-06</td>
<td>9.537E-07</td>
<td>-2.201E-04</td>
<td>6.584E-05</td>
<td>1.278E-10</td>
<td>-5.079E+04</td>
<td>2.307E+08</td>
</tr>
<tr>
<td>7.617E-06</td>
<td>9.586E-07</td>
<td>-2.221E-04</td>
<td>3.341E-05</td>
<td>8.800E-11</td>
<td>-1.989E+05</td>
<td>8.957E+08</td>
</tr>
<tr>
<td>1.714E-05</td>
<td>9.849E-07</td>
<td>-2.246E-04</td>
<td>1.536E-05</td>
<td>4.671E-11</td>
<td>-9.520E+05</td>
<td>4.239E+09</td>
</tr>
<tr>
<td>2.285E-05</td>
<td>1.010E-06</td>
<td>-2.256E-04</td>
<td>1.179E-05</td>
<td>3.450E-11</td>
<td>-1.623E+06</td>
<td>7.194E+09</td>
</tr>
</tbody>
</table>
Webster  

**Lab 5: Earth’s Magnetic Field**

<table>
<thead>
<tr>
<th>Slope ((Tm^{-1}))</th>
<th>(\sigma_{\text{Slope}}) ((Tm^{-1}))</th>
<th>(C) ((Tm^{-1}))</th>
<th>(\sigma_{C}) ((Tm^{-1}))</th>
<th>Intercept ((d_E)) ((m))</th>
<th>(\sigma_{dE}) ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.3962E+03</td>
<td>0.0331E+03</td>
<td>-2.2747E-04</td>
<td>3.426E-06</td>
<td>1.6678E-01</td>
<td>0.0131E-01</td>
</tr>
</tbody>
</table>

In the above table, the standard deviation from the average \(C\) value is roughly on the same order of magnitude as the individual uncertainties. The standard deviation acts as a good approximate uncertainty for the calibration value.

**Table 4:** The table below shows information for the graph of \(d_T\) vs. \(B_H\) given by the program LineFit. The slope is equal to \(1/C\).
Graph 1: The graph below shows the total deflection (dₜ or d_T) plotted versus the magnetic field of the Helmholtz coil (B_H or B_H). From this graph one can find the calibration constant using the slope, and the deflection caused by the Earth’s magnetic field (dₑ) is the intercept. The information plotted in this graph is from Table 2 & 3, and the information determined by the graph is in Table 4.
**Table 5 & 6:** The data shown in these two tables is the calculated values for the magnetic field components as well as the total Earth magnetic field. The magnetic fields are calculated three different ways: with the average C value, with the weighted average C, and with the slope determined value for C. $B_V$ is the vertical component, $B_\parallel$ is the parallel component, $B_\perp$ is the perpendicular component, $B_{\text{(Earth Total)}}$ is the total magnetic field, $B_{\text{(Horizontal)}}$ is the horizontal component. The table also shows each component’s uncertainty to the right of the value. The units are Tesla. For comparison and convenience, the values can be multiplied by $10^9$ for conversion to nano-Tesla.

<table>
<thead>
<tr>
<th></th>
<th>$B_V$ (T)</th>
<th>$\sigma_{B_V}$ (T)</th>
<th>$B_\perp$ (T)</th>
<th>$\sigma_{B_\perp}$ (T)</th>
<th>$B_\parallel$ (T)</th>
<th>$\sigma_{B_\parallel}$ (T)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$B_{\text{(Earth Total)}}$ (T)</th>
<th>$\sigma_{B_{\text{(Earth Total)}}}$ (T)</th>
<th>$B_{\text{(Horizontal)}}$ (T)</th>
<th>$\sigma_{B_{\text{(Horizontal)}}}$ (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. C</td>
<td>5.5229E-05</td>
<td>5.8918E-07</td>
<td>3.9383E-05</td>
<td>5.5271E-07</td>
</tr>
<tr>
<td>Wtd. Avg. C</td>
<td>5.4337E-05</td>
<td>5.6979E-07</td>
<td>3.8747E-05</td>
<td>5.3764E-07</td>
</tr>
<tr>
<td>Slope 1/C</td>
<td>5.4282E-05</td>
<td>6.5824E-07</td>
<td>3.8708E-05</td>
<td>6.0372E-07</td>
</tr>
</tbody>
</table>
Analysis

The largest part of this lab is involved in solving the equations. Understanding how each intermediate result contributes to the final is of key importance. The following paragraph is my attempt at emphasizing the stages of the calculations.

Using the recorded data, the magnetic field of the Helmholtz coil can be determined. With the Helmholtz magnetic field, the calibration constant can be determined. Using the calibration constant, the vertical, parallel, and perpendicular components of the Earth’s magnetic field can be calculated. Then the horizontal component can be determined by the parallel and perpendicular components. Finally, the total magnetic field can be determined by using the horizontal and vertical components.

We can calculate the magnetic field for the Helmholtz coil by using equation (2.9):

$$B_H = \frac{8\mu_0 NI}{a\sqrt{125}} \quad (2.9)$$

$$B_H = \frac{8(1.25663706 \times 10^{-6} \text{ m kg s}^{-2} \text{ A}^{-2})(72)(0.02 \text{ A})}{(0.34 \text{ m})\sqrt{125}} = -3.8083 \times 10^{-6} \text{ T}$$

The uncertainty in the magnetic field of the Helmholtz coil can now be calculated using equation (2.91):

$$\sigma_{B_H} = \sqrt{\left(\frac{8\mu_0 N}{a\sqrt{125}}\right)^2 \sigma_I^2 + \left(\frac{8\mu_0 NI}{a^2\sqrt{125}}\right)^2 \sigma_a^2} \quad (2.91)$$

$$\sigma_{B_H} = 9.5371 \times 10^{-7} \text{ T}$$

The calibration constant, $C$, can be calculated using equation (2.6) with the magnetic field of the Helmholtz coil determined above. The values for $d_T$ and $d_E$ are the deflection values found in Table 1 and Table 2. They must be converted to meters by dividing by 100.

$$C = B_H/(d_T - d_E) \quad (2.6)$$
\[ C = \frac{9.5371 \times 10^{-7} \ T}{(18.6 \ cm - 16.73 \ cm)} = -2.0365 \times 10^{-4} \ T \ m^{-1} \]

With the \( C \) values calculated in this way, we can find a mean (average), and also a standard deviation. For simplicities sake, I will only calculate the mean and standard deviation of two values.

\[
\text{mean} = \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i \quad (3.2)
\]

(3.2) expressed algebraically is just,

\[
\text{mean} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_N}{N}
\]

So in our case,

\[
\bar{C} = \frac{(-2.0365 \times 10^{-4} \ T \ m^{-1}) + (-2.2601 \times 10^{-4} \ T \ m^{-1})}{2} = -2.1483 \times 10^{-4} \ T \ m^{-1}
\]

The standard deviation of these two values is then,

\[
\sigma_x = \sqrt{\frac{\sum_{i=1}^{N} (\bar{x} - x_i)^2}{N - 1}} \quad (3.3)
\]

(3.3) expressed algebraically is just,

\[
\sigma_x = \sqrt{\frac{(\bar{x} - x_1)^2 + (\bar{x} - x_2)^2 + \cdots + (\bar{x} - x_N)^2}{N - 1}}
\]

\[
\sigma_C = \frac{\sqrt{[(-2.1483 \times 10^{-4} \ T \ m^{-1}) - (-2.0365 \times 10^{-4} \ T \ m^{-1})]^2 + [(-2.1483 \times 10^{-4} \ T \ m^{-1}) - (-2.2601 \times 10^{-4})]^2}}{2 - 1}
\]

\[
\sigma_C = 1.5811 \times 10^{-5} \ T \ m^{-1}
\]

Remember that this mean value and uncertainty is just for the two values shown above, and they simply serve as a means of showing the steps involved in obtaining the actual values (which is a much more lengthy procedure). In Table 2, there is a separate column for the
individual deviations from the mean. This is just a way to make the calculations easier by doing the math in parts. It also allows one to see numerically how much a certain data point differs from the mean.

The weighted average $C$ value is found by dividing the sum of the $C/\sigma_C^2$ by the sum of the $1/\sigma_C^2$ values. I will assume that the reader can do simple arithmetic division and omit these steps.

Now it’s time to calculate the Earth’s magnetic field values. We can do this by using (2.7) with simple arithmetic:

$$B_E = C d_E \quad (2.7)$$

For the vertical orientation, using the average vertical deflection and average calibration constant value:

$$B_V = (-2.314 \times 10^{-4} \ T \ m^{-1})(0.1673 \ m) = -3.8720 \times 10^{-5} \ T$$

The uncertainty can be calculated using (2.75):

$$\sigma_{B_V} = \sqrt{(0.1673 \ m)^2(2.507 \times 10^{-6} \ T \ m^{-1})^2 + (-2.314 \times 10^{-4} \ T \ m^{-1})^2(0.002 \ m)^2} = 6.2468 \times 10^{-6} \ T$$

Now with the two previous calculations accomplished, the same goes for the parallel and perpendicular components and their uncertainties.

For the horizontal component of the Earth’s magnetic field we must use (3.0), and for its uncertainty we shall use (3.01). For both calculations I will use the average C magnetic field values:

$$B_{h} = \sqrt{B_{\|}^2 + B_{\perp}^2} \quad (3.0)$$

$$B_{h} = \sqrt{(2.7166 \times 10^{-5} \ T)^2 + (-2.8513 \times 10^{-5} \ T)^2} = 3.9383 \times 10^{-5} \ T$$

$$\sigma_{B_{h}} = \sqrt{\frac{B_{\|}^2 \sigma_{B_{\|}}^2 + B_{\perp}^2 \sigma_{B_{\perp}}^2}{B_{\|}^2 + B_{\perp}^2}} \quad (3.01)$$
\[ \sigma_{B_h} = \sqrt{(2.7166 \times 10^{-5} T)^2 (5.4852 \times 10^{-7})^2 + (-2.8513 \times 10^{-5} T)^2 (5.5649 \times 10^{-7})^2} \]

\[ (2.7166 \times 10^{-5} T)^2 + (-2.8513 \times 10^{-5} T)^2 \]

\[ \sigma_{B_h} = 5.5271 \times 10^{-7} T \]

With the horizontal component and its uncertainty calculated (along with the vertical component and its uncertainty), now the total magnetic field of the Earth and its uncertainty can be calculated:

\[ B_{Earth\, Tot.} = \sqrt{B_h^2 + B_v^2} \quad (3.1) \]

\[ B_{Earth\, Tot.} = \sqrt{(3.9383 \times 10^{-5} T)^2 + (-3.8720 \times 10^{-5} T)^2} = 5.5229 \times 10^{-5} T = 55228.6824 \, nT \]

\[ \sigma_{B_{Earth\, Tot.}} = \sqrt{\frac{B_h^2 \sigma_{B_h}^2 + B_v^2 \sigma_{B_v}^2}{B_h^2 + B_v^2}} \quad (3.11) \]

\[ \sigma_{B_{Earth\, Tot.}} = \sqrt{(3.9383 \times 10^{-5} T)^2 (5.5271 \times 10^{-7} T)^2 + (-3.8720 \times 10^{-5} T)^2 (6.2468 \times 10^{-6} T)^2} \]

\[ (3.9383 \times 10^{-5} T)^2 + (-3.8720 \times 10^{-5} T)^2 \]

\[ \sigma_{B_{Earth\, Tot.}} = 5.8918 \times 10^{-7} T = 589.1813 \, nT \]

This calculated value of the total magnetic field of the Earth and uncertainty is on the same order of magnitude as the accepted value, though the magnitude differs by a larger degree than the uncertainty permits.
Discussion

The determined value for the total magnetic field of the Earth is on the same order as the accepted value (47655 nT on the 18th of November in Tallahassee, FL), but it is off by around 14%. The uncertainty also does not bridge this gap of 7574 nT (our most accurate value determined5 results in a gap of 6627 nT), which hints to something possibly wrong in the calculations, but I was unable to pinpoint the source of this error. Since the vertical component is close to its accepted value of 41194 nT that leads me to believe the horizontal component is where the problem lies. For the horizontal component value determined previously in the Analysis section, it is on the same order of magnitude as the accepted value, but much larger (by roughly 39%). The values that go into determining the horizontal component are the parallel and perpendicular components. This means that either the value for the calibration constant is wrong, or there is something wrong with the average deflection value being used. Since the calibration constant affects the vertical, parallel, and perpendicular components in the same way I doubt that is where the major source of error in the horizontal component resides. This has led me to believe that one or more of the average deflection values are incorrect. This could be due to incorrect alignment of the flip coil. In addition, the vertical and horizontal components accepted values are both positive, however it was determined in this experiment that the vertical component is negative. Also, examining the accepted values for the vertical and horizontal components shows that the horizontal should have been roughly one-half of the vertical components intensity.

Table 6:6 The accepted values for the components of the Earth’s magnetic field for Tallahassee, FL on November 18th, 2013. The “Horizontal Intensity” is the horizontal component, the “North Comp” is the parallel component, the “East Comp” is the perpendicular component, the “Vertical Comp” is the vertical component, and the “Total Field” is the Earth’s total magnetic field. Units are in nano-Tesla (nT or $10^{-9}$ T).

| Date       | Declination (+ E | - W) | Inclination (+ D | - U) | Horizontal Intensity | North Comp (+ N | - S) | East Comp (+ E | - W) | Vertical Comp (+ D | - U) | Total Field       |
|------------|------------------|---------|------------------|----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|------------------|
| 2013-11-18 | -4° 23' 39"     | 59° 49' 3" | 23,958.9 nT      | 23,888.4 nT          | -1,835.6 nT     | 41,194.3 nT     | 47,655.0 nT     |
| Change/year| -6.2°            | -5.3'    | -1.4 nT          | -4.8 nT              | -42.9 nT        | -147.0 nT       | -127.8 nT       |

The above accepted values show us that there is a systematic error in our experiment. The error arises from the deflection values recorded in the lab. This is more than likely due to improper alignment of the flip coil axis. If the deflection value is negative, the magnetic field will be positive (since the calibration constant is negative). Therefore, the perpendicular

5 In which the calibration constant that was determined by the slope of the graph was used.

6 (NOAA)
component deflection values should be the only positive deflections. Since ours were recorded as being positive, this is a systematic error (a flaw in the experimental procedure).

A mis-measurement of the Helmholtz coil radius would also increase the error in the determined values for the Earth’s magnetic field. An increase in the radius would make the Helmholtz magnetic field value smaller, which in turn would make the calibration constant larger. The larger calibration constant would cause the magnetic field component to become larger. The overall consequence of a larger radius would decrease the value for the total Earth magnetic field. For roughly double the radius, the value for the total Earth magnetic field decreases by roughly a half. A mis-measurement resulting in a smaller value would just be the opposite of this. I do not believe that there was a mis-measurement of the Helmholtz coils radius in the case of our experiment. As I am not able to return to the lab currently I will use reasoning to gain a better understanding of our radius. Our radius was determined by measurement to be 0.34 ± 0.005 m, which seems like an acceptable radius. Converting 0.34 m to feet (a unit I am more likely to “eyeball” correctly) gives roughly 1 foot. If memory of the coil size serves me correctly, 1 foot definitely seems justifiable.
Conclusion

This experiment sought to determine the Earth’s total magnetic field. A calibration constant was using the known magnetic field of a Helmholtz coil. This calibration constant was applied to deflection values obtained by observing the effects of a flip coil through a ballistic galvanometer. The determined values for the vertical, perpendicular, and parallel components of the Earth’s magnetic field are, respectively: \(-38056 \pm 732 \, \text{nT}, -28024 \pm 521 \, \text{nT}, 26700 \pm 607 \, \text{nT}\). The horizontal component of the Earth’s magnetic field was determined to be \(38708 \pm 604 \, \text{nT}\), and finally the total magnetic field of the Earth was determined to be \(54282 \pm 658 \, \text{nT}\).
Appendix

A.1 Formula for the propagation of errors:

Given a function, $f$, with variables $x$, $y$, and $n$. The uncertainty in $f$ is the square root of the sum of the squares of the partial derivatives of $f$ with respect to each variable, and each partial derivative is multiplied by the square of its uncertainty.

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \ldots + \left(\frac{\partial f}{\partial n}\right)^2 \sigma_n^2} \quad (A.1)$$
References


*The Earth Inductor, or Delzenie's Circle.* (n.d.). Retrieved November 21, 2013, from Kenyon College: http://physics.kenyon.edu/EarlyApparatus/Electricity/Earth_Inductor_or_Delzennes_Circle/Earth_Inductor_or_Delzennes_Circle.html